Cambridge IGCSE™

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

CAMBRIDGE INTERNATIONAL MATHEMATICS   **0607/51**

Paper 5 Investigation (Core)   **May/June 2024**

1 hour 10 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

* Answer **all** questions.
* Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
* Write your name, centre number and candidate number in the boxes at the top of the page.
* Write your answer to each question in the space provided.
* Do **not** use an erasable pen or correction fluid.
* Do **not** write on any bar codes.
* You should use a graphic display calculator where appropriate.
* You may use tracing paper.
* You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
* In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

* The total mark for this paper is 36.
* The number of marks for each question or part question is shown in brackets [ ].

This document has **8** pages. Any blank pages are indicated.

Answer **all** the questions.

INVESTIGATION   **SUMS OF NUMBERS**

This investigation looks at the sum of the positive integers, $1 + 2 + 3 + \cdots$.

It also looks at the sum of their cubes, $1^3 + 2^3 + 3^3 + \cdots$.

1 **(a)** Calculate $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$.

.... [1]

(b) The numbers can also be added using this method.

Complete this method for the sum of the numbers in **part (a)**.

Step 1	Write down the first half of the numbers in a row.	1 2 3 4
Step 2	Write down the second half of the numbers underneath the first half but in reverse order.	8 7 6 5
Step 3	Add each column of two numbers to make a third row.	9 9 9 9
Step 4	Find the total of the numbers in the third row by writing the calculation as a multiplication.	$4 \times 9 = $ ....

[1]

2 Use the method in **Question 1(b)** to calculate the sum of the first six positive integers, $1 + 2 + 3 + 4 + 5 + 6$.

[4]

3 Complete the method to show that the sum of the first 30 positive integers is 465.

So $1 + 2 + 3 + \cdots + 28 + 29 + 30 = 465$.

1	2	3	$\cdots$	15
....	....	....	$\cdots$	16
....	....	....	$\cdots$	....

$$............... \times .... \quad = \quad 465 \qquad [3]$$

4 Find the sum of the first 160 positive integers, $1 + 2 + 3 + \cdots + 160$.

.... [3]

 [Turn over

5 Use **Questions 1**, **2**, **3** and **4** to help you complete the table.

	Number of positive integers starting at 1	Multiplication	Sum
	4	2×5	10
Question 2	6		
Question 1	8	4×9	
	10		
Question 3	30		465
	50	25×51	1275
	128	64×129	
Question 4	160		
	204		20910

[4]

6 $1 + 2 + 3 + \cdots + n$ has n positive integers and its sum is T.

Find a formula for T in terms of n.

.... [3]

7 In the table in **Question 5** the number of integers is always even.

Does your formula in **Question 6** give the correct total when $n = 7$?

[3]

8 **(a)** Complete the table.

Sum of first n positive integers		Sum of first n cube numbers			
Calculation	Sum (T)			Calculation	Sum (S)
$1+2$	3	1^3+2^3	$1+8$		9
$1+2+3$		$1^3+2^3+3^3$			36
$1+2+3+4$			$1+8+27+64$		
	15	$1^3+2^3+3^3+4^3+5^3$			225

[4]

(b) **(i)** Write down the mathematical name for the positive integers in the Sum (S) column.

.... [1]

(ii) Write a formula for S in terms of T.

.... [1]

(c) Use your answers to **Question 6** and **Question 8(b)** to find the sum of the cubes of the first 40 positive integers.

$$1^3 + 2^3 + 3^3 + \cdots + 40^3$$

.... [4]

(d) $1^3 + 2^3 + 3^3 + \cdots + n^3 = 396\,900$

Find how many cube numbers there are in this sum.

.... [4]

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